10.6 Models

Some models we've seen:

$y' = ky$ "rate of change is proportional to the current amount"

- Unrestricted population growth
- Interest: growth of money
- Radioactive decay

$y' = k(M - y)$ "rate of change is proportional to the difference in M and the current amount"

- Velocity of a sky diver with air resistance.
- Newton's Law of Cooling.

Logistic model for population growth in ecology.

$$\frac{dN}{dt} = rN\left(\frac{K-N}{K}\right)$$

$$\frac{dN}{dt} = \frac{r}{K}N(K-N)$$

N: # of individuals in population

K: carrying capacity.

r: Intrisic growth rate

If our population $0 \leq N \leq k$

then $0 \leq \frac{K-N}{K} \leq 1$

$\frac{K-N}{K} = 1$ ↑ close to 1 ... then $\frac{dN}{dt} = rN$ ↑ close to

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A population of fish in a ~~lake~~ lake.

we have a carrying capacity of 1000 fish

and ~~r = 0.1~~ $r = 0.2$

~~Using~~ Sketch population curves of the following scenarios:

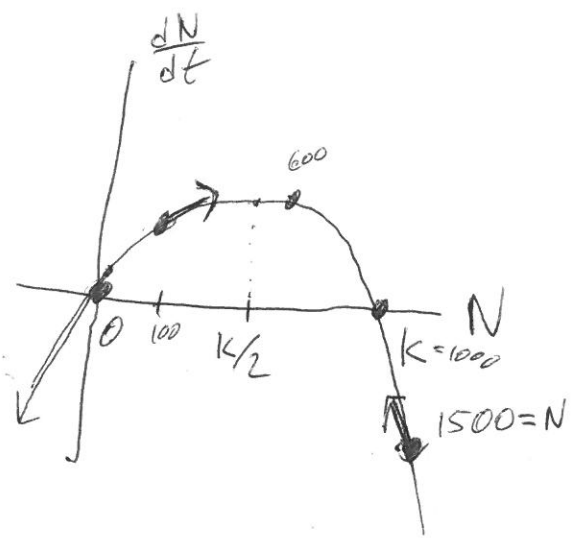
(1) What will happen w/ initial population of 100 fish?

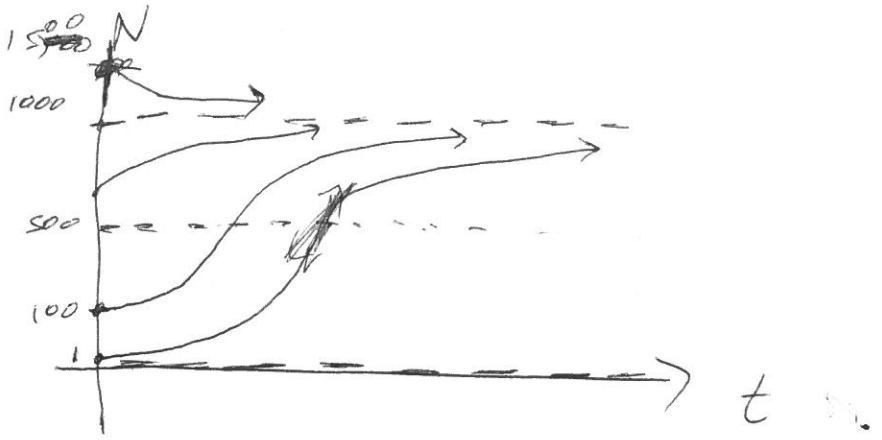
(2) An initial population of 1,500 fish?

(3) An initial population of 1 fish?

$$\frac{dN}{dt} = \frac{r}{k} N(K-N)$$

$$= \frac{0.2}{1000} N(1000-N)$$





If we start with 600 fish

Q: What population of fish has the fastest growth rate and what is it?
(t measured in months)

$$\frac{dN}{dt} = \frac{0.2}{1000}(1000 - N)$$

Max growth rate happens when $N = 500$

Max growth rate is:

$$\left.\frac{dN}{dt}\right|_{N=500} = \frac{.2}{1000}(1000 - 500)$$

$$= \frac{.2}{1000}(500)(500)$$

$$= \frac{1}{5}\cdot\frac{5}{1000}\cdot 500 = \frac{500}{10}$$

$$= 50 \ \frac{\text{fish}}{\text{month}}$$

You are a detective in a crime scene.

Find a cup of tea at 180°F

You know that this kind of tea uses boiling water at 212°F.

10 mins later the tea is 150°F making the current time 7:10 AM.

Q when was this tea brewed?

Q2. You then remember you are in Denver, CO at 5,000 ft ~~the~~ water boils at 203°F. when was the tea brewed?

* You also remember that Newton's law of cooling says the tea cools at a rate proportional to the difference between the current temp and room temp (70°F).